The Swimming of Manta Rays

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Aquatic animals propel themselves using a wide variety of mechanisms. In manta rays, propulsion is achieved by combining oscillating and undulatory motions of flexible surfaces. We are interested in studying the unsteady hydrodynamics of such motions to understand and model the wake structure. Experiments suggest a rich set of phenomena exist, depending on the non-dimensional frequency of flapping, the wavelength of the excitation, and the aspect ratio of the fin. Under certain conditions, simple wake structures are observed that bear a strong resemblance to the structure of co-flowing jets and wakes. In other cases, bifurcating wakes are seen, and both cases appear to correspond to a peak in efficiency.

We describe two principal sets of experiments: undulating and flapping fins of elliptical planform, and pitching panels of rectangular planform with varying flexibility. To interpret the results on thrust and efficiency, we propose scalings for aspect ratio and flexibility, and develop a stability analysis called wake resonance theory. Here we focus on the insights provided by wake resonance theory.

The motivating result is shown in Figure 1. Here, we present the efficiency η_p of a mechanical analog of a manta ray pectoral fin, actuated using four rigid spars to produce an undulatory motion. Two observations are important: the peak efficiency exceeds 50%, and there are two peaks in efficiency, one at a Strouhal number St = 0.2 corresponding to a 2S wake (two single vortices are shed per flapping cycle), and one at a Strouhal number of 0.3 corresponding to a 2P wake (two pairs of vortices are shed per flapping cycle). Here, St = fA/U, where f is the frequency of actuation, A is the peak-to-peak amplitude of the trailing edge motion at the half-span, and U is the freestream velocity.

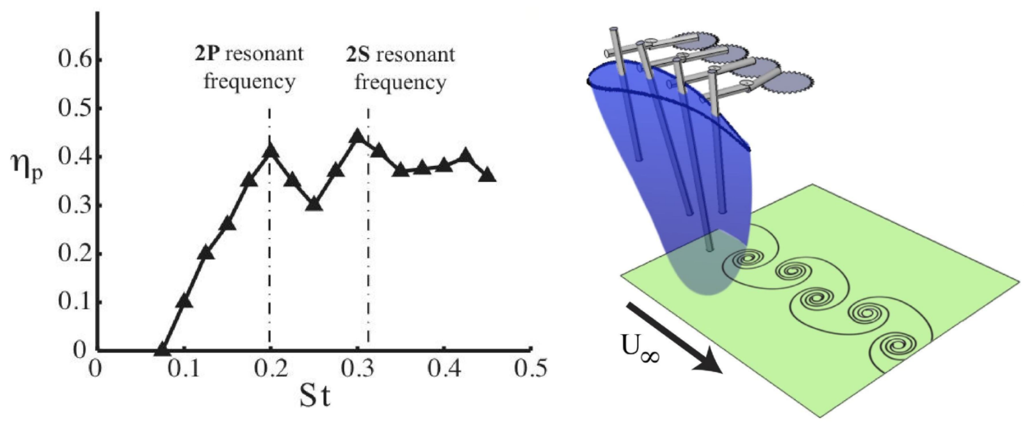

Figure 1: Left: Efficiency of a batoid-like fin actuated in undulatory motion with a wavelength four times the chord length (adapted from Clark & Smits, 2008). Right: Fin schematic, showing 2S wake.

To provide insight into this behavior, we can examine the stability of the time-averaged wake profile. A linear spatial stability analysis is used to find the frequency of maximum spatial growth, that is, the hydrodynamic resonant frequency of the time-averaged jet. The details are given by Moored et al. (2012a). It is found that: (i) optima in propulsive efficiency occur when the driving frequency of a flapping fin matches the resonant frequency of the jet profile; (ii) there can be multiple wake resonant frequencies and modes corresponding to multiple peaks in efficiency; and (iii) some wake structures transition from one pattern to another when the wake instability mode transitions. The results are illustrated in Figure 2, which shows the eigenvalues at a given forcing frequency. It is postulated that when the most unstable eigenvalue of the wake profile coincides with the driving frequency of the fin there is a peak in efficiency. The dashed lines in Figure 1 correspond to the two cases shown in Figure 2, and indeed there is a close correspondence. The theoretical framework is termed wake resonance theory, and the analysis, although one-dimensional, captures the performance exhibited by a three-dimensional propulsor, showing the robustness and broad applicability of the technique.

Dewey et al. (2012) and Moored et al. (2012b) applied the same analysis to the flowfields generated by flexible fins of rectangular planform. Here, the efficiency peaks in a global sense when the structural resonant frequencies of the panels are nearly aligned with their wake resonant frequencies.

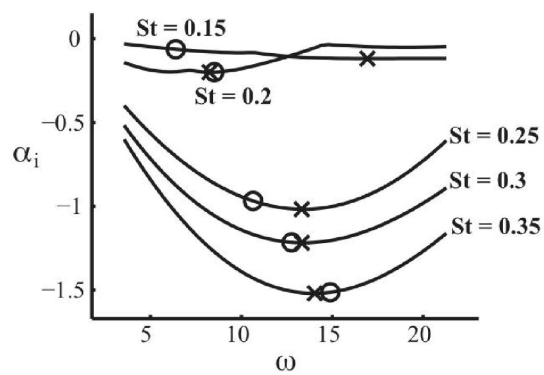

Figure 2: Stability curves for five velocity profiles taken from PIV data measured by Dewey et al. (2011). The × mark the resonant frequency of a stability curve while the o mark the driving frequency used to generate the velocity profile.

References

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